

MATHEMATICS

Study Guide- Specialization Test
(Grade 9 – Grade 12) Teachers

Year 2020

Table of Contents

Test Overview.....	Page 3
Test specifications.....	Page 4
Sample Questions	Page 8
Answer Key.....	Page 29

Educational Professions Licensure

Mathematics Study Guide

The Teacher Licensing System in the Ministry of Education of the United Arab Emirates is one of the educational priorities that aim at optimizing investment in teachers in order to help them to achieve the objectives of the ministry and to improve educational outcomes.

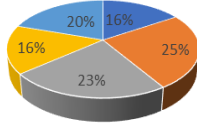
The Mathematics Test for teachers is one of the Professional teacher's license requirements for those who are teaching from grades 9 through 12 in mathematics.

Test Overview

Test Name	Mathematics Test
Number of questions	100
Test Duration	2 and ½ hours
Format of questions	Multiple Choice questions/ Fill in the blank
Test Delivery	Computer delivered

Content Domain	Approximate Percentage of Test	Approximate Number of Questions
I. Number and Quantity	16%	16
II. Algebra	25%	25
III. Geometry and Measurement	23%	23
IV. Probability and Statistics	16%	16
V. Trigonometry, Calculus, and Discrete	20%	20

Approximate % of Test Domains



- I. Number and Quantity
- II. Algebra
- III. Geometry and Measurement
- IV. Probability and Statistics
- V. Trigonometry, Calculus, and Discrete

Test specifications

I. Number and Quantity

1. Structure of Numerical Systems:

- Place value
- Order relationships
- Relationships between operations
- Multiple forms of numbers
- Absolute value
- Signed numbers
- Integers and rational numbers
- Ratios and proportion

2. Real and Complex Number Systems:

- Rational and irrational numbers
- Multiple forms of complex numbers
- Properties of the real and complex number systems
- Operations with complex numbers
- Laws of exponents
- Roots and powers of real and complex numbers
- Scientific notation

3. Elementary Number Theory:

- Factors and divisibility
- Prime and composite
- Prime Factorization
- Euclid's Algorithm
- Congruence classes and modular arithmetic
- Mersenne primes and perfect numbers
- Fermat's Last Theorem
- Fundamental Theorem of Arithmetic

II. Algebra

1. Patterns and Modeling:

- Patterns in numeric, geometric, or tabular form
- Symbolic notation
- Patterns created by functions
- Iterative and recursive functional relationships
- Pascal's triangle and binomial theorem
- Finite and infinite sequences and series

2. Functions and Relations:

- Differences between functions and relations
- Multiple forms of functions
- Properties of functions and relations
- Piecewise, composite, and inverse functions
- Graphs of functions and their transformations

3. Linear Functions and Relations:

- Linear models and rates of change
- Direct variation
- Graphs of linear functions
- Slopes and intercepts of lines
- Equations of lines and inequalities
- Expressions involving absolute value
- Solve problems involving linear functions and systems.

II. Algebra cont.

4. Application of linear and abstract algebra:

- Properties of matrices and determinants
- Solving linear systems using matrices
- Geometric and algebraic properties of vectors
- Properties of vector spaces
- Matrix representation of linear transformation
- Definitions and properties of groups, rings, and fields

5. Quadratic Functions and Relations:

- Simplification of quadratic expressions
- Solving quadratic equations and inequalities
- Real and complex roots of quadratic equations
- Graphs of quadratic equations
- Graphical and symbolic representation of quadratic functions
- Maximum and minimum problems
- Modeling with quadratic relations, functions, and systems

6. Polynomial, Rational, Radical, and Absolute Value Functions and Relations:

- Inverse and joint variations
- Zeros of polynomial functions
- Simplifying polynomial and rational expressions
- Horizontal, vertical, and slant asymptotes
- Solving problems involving polynomial, rational, radical, absolute value, and step functions

7. Logarithmic and Exponential Functions and Relations:

- Simplifying logarithmic and exponential expressions
- Properties of logarithmic and exponential functions
- Applications involving exponential growth, decay, and compound interest
- Inverse relationships between logarithmic and exponential functions

III. Geometry and Measurement

1. Structure of Euclidean Geometry:

- Axiomatic systems
- Undefined terms, postulates, and theorems
- Relationships between, points, lines, rays, angles, and planes
- Axioms of algebra
- Distance and angle measure postulates
- Special pairs of angles
- Properties of parallel and perpendicular lines and planes
- Triangle congruence conditions
- Pythagorean theorem
- Segments and angles associated with circles

2. Proofs and Theorems:

- Direct and indirect proofs
- Properties of special triangles
- Characteristics of parallelograms and quadrilaterals
- Similar triangles and other polygons
- Geometric constructions
- Theorems and properties of circles

III. Geometry and Measurement cont.

3. Two- and Three-Dimensional Objects:

- a. Special right triangle relationships
- b. Arcs, angles, segments associated with polygons and circles
- c. Properties of three-dimensional figures
- d. Perspective drawings and projections
- e. Cross-sections and nets
- f. Deriving properties of three-dimensional objects from two-dimensional shapes
- g. Modeling involving three-dimensional objects

4. Coordinate and Transformational Geometry:

- a. Rectangular and polar coordinates
- b. Geometric figures in the coordinate plane
- c. Three-dimensional coordinate systems
- d. Characteristics of distance, midpoint, slope, and parallel and perpendicular lines for problem solving
- e. Translations, rotations, reflections, glide-reflections and dilation
- f. Types of symmetry
- g. Axioms and features of non-Euclidean geometries

IV. Probability and Statistics

1. Descriptive Statistics and Data:

- a. Charts, graphs, and tabular data representations
- b. Determine appropriate sampling techniques and gathering data
- c. Designing statistical experiments
- d. Inferences about a population from sample statistics

2. Measures:

- a. Measures of central tendency
- b. Dispersion
- c. Frequency distributions
- d. Percentile scores
- e. Effects of data transformations on measure of central tendency and variability
- f. Effects of sample size
- g. Interpreting correlation
- h. Problems involving regression models and the line of best fit

3. Probability:

- a. Probabilities of simple and compound events
- b. Simulations to analyze probability
- c. Probability as a ratio of two areas
- d. Random variables and probability distributions

V. Trigonometry, Calculus, and Discrete Mathematics

1. Discrete Mathematics:

- a. Properties of sets
- b. Counting techniques
- c. Finite Differences
- d. Mathematics of finance
- e. Recursive patterns and relations
- f. Iteration
- g. Linear programming in two variables
- h. Properties of Matrices
- i. Finite graphs and trees

2. Trigonometry:

- a. Degree and radian measure
- b. Right triangle trigonometry
- c. Law of sines and cosines
- d. Relationship between unit circle and trigonometric functions
- e. Properties and graphs of trigonometric functions and inverses
- f. Trigonometric identities
- g. Trigonometric equations

3. Calculus - Limits:

- a. Limits of algebraic functions
- b. Limits of infinite sequences and series
- c. Continuous and discontinuous functions
- d. Relationship between the secant line and the average rate of change of a function
- e. Problems involving average rates of change

V. Trigonometry, Calculus, and Discrete Mathematics cont.

4. Calculus - Derivatives:

- a. Slope of the line tangent to a curve
- b. Properties and definition of the derivative
- c. Differentiability
- d. Techniques of differentiation
- e. Derivatives of algebraic and transcendental functions
- f. Application using differentiation
- g. Verify given functions as a solution of differential equations

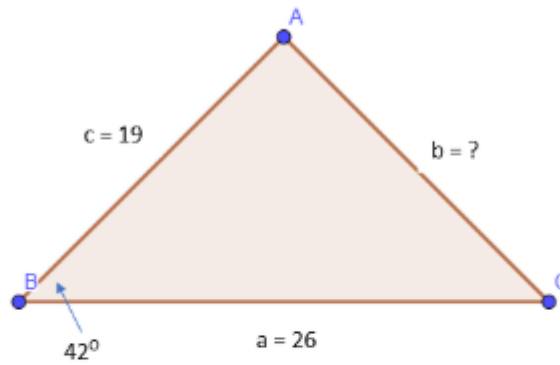
5. Calculus - Integrals:

- a. Algebraic and geometric approximations of the area under a curve
- b. The integral as the limit of a Riemann sum
- c. Fundamental Theorem of Calculus
- d. Techniques of integration
- e. Applications using integration
- f. Solving differential equations by separation of variables

Sample Questions

1. What is the length of side b for the triangle shown below?

ما طول الضلع b في المثلث الموضح أدناه؟



- A.
- B.
- C.
- D.

2. What is the standard deviation for the data set shown below? ما الإنحراف المعياري لمجموعة البيانات الموضحة أدناه؟

{1, 3, 5, 7}

- A.
- B.
- C.
- D.

3. The radius of a sphere is increasing at a rate of 6 cm/s. How fast is the volume increasing when the diameter is 40 cm? يتزايد طول نصف قطر كرة بمعدل 6 cm/s ما معدل تزايد الحجم عندما يكون القطر 40 cm؟

- A.
- B.
- C.
- D.

4. Simplify the expression below.

بسّط التعبير أدناه.

$$(2 + 2i)(2 - 2i)$$

- A.
- B.
- C.
- D.

5. An investment of AED 2000 is made into an account with an annual interest rate of 5%, compounded continuously. What is the total value for the investment after eight years?

في حساب ما أستثمر مبلغ AED 2000 بمعدل فائدة سنوية مركبة قدرها 5%. ما إجمالي المبلغ المُستثمر بعد ثمان سنوات؟

- A.
- B.
- C.
- D.

6. A recursively defined sequence is shown below.

متتالية تكرارية معرفة كما يلي

$$a_1 = 9, a_n = a_{n-1} + 6$$

Which of the following is an explicit formula that represents the same sequence of numbers?

أي من العلاقات الصريحة التالية تمثل نفس المتتالية؟

- A.
- B.
- C.
- D.

7. What are the zeros of the function shown below?

ما أصفار الدالة الموضحة أدناه؟

$$f(x) = x^3 + 4x^2 + 4x$$

- A.
- B.
- C.
- D.

8. Which of the following functions forms an equivalent graph to the function shown below?

أي دالة من الدوال التالية لها منحنى يكافئ منحنى الدالة الموضحة أدناه؟

$$y = \cos(x)$$

- A.
- B.
- C.
- D.



9. For group of 20 men, the median weight is 85 kilograms with a range of 14 kilograms. If each man gains 5 kilograms, which of the following would be true?

مجموعة من 20 رجلاً، الوسيط لأوزانهم يساوي 85 كيلوجرام والمدى 14 كيلوجرام. إذا زاد وزن كل رجل 5 كيلوجرامات، أي مما يلي سوف يكون صحيحاً؟

- A. The median weight will stay the same, and the range will increase. الوسيط سيبقى نفسه والمدى سيزداد.
- B. The median weight will increase and the range will remain the same. الوسيط سيزداد والمدى سيبقى نفسه.
- C. The median weight and range will both remain the same. الوسيط والمدى كلاهما سيبقى نفسه.
- D. The median weight and range will both increase. الوسيط والمدى كلاهما سيزداد.



10. Does the series shown below converge or diverge? If convergent, find its sum.

هل المتسلسلة الموضحة أدناه متقاربة أو متباعدة؟ إذا متقاربة، أوجد مجموعها؟

$$\sum_{k=0}^{\infty} (-1)^k \left(\frac{2}{3}\right)^k$$

- A. convergent, $\frac{5}{3}$ متقاربة، $\frac{5}{3}$
- B. convergent, $\frac{3}{5}$ متقاربة، $\frac{3}{5}$
- C. convergent, $\frac{2}{3}$ متقاربة، $\frac{2}{3}$
- D. divergent متباعدة.



11. A particle moves along the x -axis so that at any time $t \geq 0$, its velocity is given by the equation below.

يتحرك جسيم على طول المحور الأفقي x بحيث في أي وقت $t \geq 0$ ، سرعة الجسيم مُعطاه بالمعادلة أدناه.

$$v(t) = \frac{6}{t+3}$$

What is the acceleration of the particle at time $t = 5$?

ما تسارع الجسيم عند اللحظة $t = 5$ ؟

- A.
- B.
- C.
- D.

12. Let $f(x)$ be the function shown below.

ألتكن $f(x)$ الدالة الموضحة أدناه.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$$

Which of the following statement(s) is/are true?

أي من العبارة (العبارات) التالية صحيحة؟

1. $\lim_{x \rightarrow 2} f(x)$ exists

1. $\lim_{x \rightarrow 2} f(x)$ موجودة

2. $f(2)$ exists

2. $f(2)$ موجودة

3. f متصلة عند $x = 2$

3. f is continuous at $x = 2$

A.

1 and 2

B.

1 and 3

C.

1 only

D.

2 only

13. What is the solution to the equation shown below?

ما مجموعة حل المعادلة الموضحة أدناه؟

$$\frac{x^2 + x - 30}{x - 5} = 11$$

- A.
- B.
- C.
- D.



14. Which of the following is an equivalent expression to the expression shown below, if x is not zero?

أي تعبير مما يلي يكافئ التعبير الموضح أدناه حيث x لا تساوي صفراً؟

$$\frac{3}{x} + \frac{5u}{2x} - \frac{u}{4}$$

A.

$$\frac{3 + 5u - u}{x}$$

B.

$$\frac{12 + 10u - ux}{4x}$$

C.

$$\frac{12x + 10u + ux}{4x}$$

D.

$$\frac{12 + 10u - u}{4x}$$



15. Which of the following is equivalent to the expression below after it is simplified?

أي مما يلي يكافئ التعبير أدناه بعد التبسيط؟

$$\frac{4a^{-1}b^3}{a^4b^{-2}} \times \frac{3a}{b}$$

- A.
- B.
- C.
- D.

16. What is the most general antiderivative of the function shown below?

ما المشتقة العكسية (الدالة الأصلية) للدالة الموضحة أدناه؟

$$g(x) = \frac{1 + x + x^2}{\sqrt{x}}$$

A.

$$G(x) = x^{\frac{1}{2}} + x^{\frac{3}{2}} + x^{\frac{5}{2}} + c$$

B.

$$G(x) = x^{\frac{1}{2}} + x^{\frac{3}{2}} + x^{\frac{5}{2}}$$

C.

$$G(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}}$$

D.

$$G(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$$

17. Two cards are drawn from a shuffled deck of 52 cards. What is the probability that both cards are Kings if the first card isn't replaced after it's drawn a King?

تم سحب بطاقتين من مجموعة أوراق اللعب البالغ عددها 52 بطاقة. ما احتمال أن تكون كلا البطاقتين ملك إذا لم يتم إرجاع البطاقة الأولى بعد سحبها؟

- A.
- B.
- C.
- D.

18. How could the following equation be factored to find the zeros?

كيف يمكن أن تحلل المعادلة التالية لإيجاد الأصفار؟

$$0 = x^3 - 3x^2 - 4x$$

- A.
- B.
- C.
- D.

19. What is $\cos \frac{\pi}{8}$ evaluated exactly?

ما قيمة $\cos \frac{\pi}{8}$ بالضبط؟

- A.
- B.
- C.
- D.

20. What are the center and radius of a circle for the equation shown below? المعادلة أدناه معادلة دائرة. ما مركزها ونصف قطرها؟

$$4x^2 + 4y^2 - 16x - 24y + 51 = 0$$

- A. center (2, 3) and radius $\frac{1}{2}$ المركز (2, 3) ونصف القطر $\frac{1}{2}$
- B. center (3, 2) and radius $\frac{1}{2}$ المركز (3, 2) ونصف القطر $\frac{1}{2}$
- C. center (2, 3) and radius $\frac{1}{4}$ المركز (2, 3) ونصف القطر $\frac{1}{4}$
- D. center (3, 2) and radius $\frac{1}{4}$ المركز (3, 2) ونصف القطر $\frac{1}{4}$

21. What is the inverse of the function shown below?

ما الدالة العكسية للدالة الموضحة أدناه؟

$$f(x) = 3x - 5$$

A. $f^{-1}(x) = \frac{x}{3} + 5$

B. $f^{-1}(x) = \frac{x+5}{3}$

C. $f^{-1}(x) = \frac{5x}{3}$

D. $f^{-1}(x) = 3x + 5$

22. The function shown below satisfies the Mean Value Theorem on the interval given.

الدالة أدناه تحقق نظرية القيمة الوسطية في الفترة المعطاة.

$$f(x) = (x - 2)^3 \text{ on } [0, 2]$$

What value of c satisfies the theorem?

ما قيمة c التي تحقق النظرية؟

A.

$$-2 - \frac{2\sqrt{3}}{3}$$

B.

$$2 - \frac{2\sqrt{3}}{3}$$

C.

$$2$$

D.

$$0$$



23. What is the probability of rolling a 6 at least once in two rolls of a standard six sided die?

ما احتمال ظهور العدد 6 على الأقل مرة واحدة في رميتين لحجر نرد ذي ستة أوجه؟

A. $\frac{11}{36}$

B. $\frac{1}{3}$

C. $\frac{1}{36}$

D. $\frac{1}{6}$

24. What are the first four terms of the sequence shown below?

ما أول أربعة حدود من المتتالية الموضحة ادناه؟

$$\left\{ \frac{(-1)^{n+1}}{n^2 + 5} \right\}_{n=0}^{\infty}$$

A. $\frac{1}{6}, \frac{-1}{9}, \frac{1}{14}, \frac{-1}{19}$

B. $\frac{-1}{5}, \frac{1}{6}, \frac{-1}{9}, \frac{1}{14}$

C. $\frac{1}{5}, \frac{1}{6}, \frac{1}{9}, \frac{1}{14}$

D. $\frac{1}{6}, \frac{1}{9}, \frac{1}{14}, \frac{1}{19}$

25. What is the definite integral that represents the area of the region bounded by the graphs of the functions shown below? ما التكامل المحدد الذي يمثل مساحة المنطقة المحددة بالدوال الموضحة أدناه؟

$$y_1 = 5 - x^2$$
$$y_2 = -3x - 5$$

A. $\int_{-2}^5 [(5 - x^2) + (-3x - 5)] dx$

B. $\int_{-\sqrt{5}}^{\sqrt{5}} (5 - x^2) dx$

C. $\int_{-\sqrt{5}}^{\sqrt{5}} (-x^2 + 3x - 10) dx$

D. $\int_{-2}^5 (-x^2 + 3x + 10) dx$



Answer Key

Question	Answer
1.	D
2.	D
3.	C
4.	A
5.	B
6.	D
7.	C
8.	C
9.	B
10.	B
11.	B
12.	A
13.	D
14.	B
15.	C
16.	D
17.	D
18.	B
19.	B
20.	A
21.	B
22.	B
23.	A
24.	B
25.	D